Suppose we are given a language L consisting of a binary predicate R, a

unary predicate P and the individual constants a, b, c.

* R(P(x), y)
* ∃x(P(x) ∧ R(x, x))
* ∃x(R(x, y) ∧ P(x))
* P(a)
* R(a, x) → P(c)
* ∀x∀y(R(x, y) → R(y, x))

which ones are grammatically correct? which ones are sentences? which

ones are open? which ones are ground? which ones are universal?

* R(P(x), y)

Is not correct, R is a predicate and can not have another predicate P as an element

* ∃x(P(x) ∧ R(x, x))

It is correct. A sentence is a formula when no free variables occur, so it is a sentence. Since there is an existential quantifier the formula is not open. The formula is not ground since there is x that is a variable. Since there is no universal quantifiers, the formula is not universal

* ∃x(R(x, y) ∧ P(x))

It is correct. A sentence is a formula when no free variable occurs. So is not a sentence because of y. A formula is open if there is no quantifiers, but there is ∃, so the formula is close. a formula is ground if there are no variables, so it is not ground. The formula is not universal since there is no universal quantifier

* P(a)

The formula is grammatically correct. A sentence is a formula where no variables occur, so it is a sentence. Is open because there is no quantifier. Is ground because there is no variable and it is not universal

* R(a, x) → P(c)

The formula is grammatically correct. A formula is a sentence if no variables occur free, so this formula is not a sentence. A formula is open if there is no existential quantifier, so it is open. Is not ground because there is a variable x. Is not universal because there is no universal quantifier

* ∀x∀y(R(x, y) → R(y, x))

The formula is grammatically correct. A sentence is a formula when no free variables occur, so it is a sentence. It is close since there are quantifiers. It is not ground because there are variables. It is universal because there are two universal quantifiers

# Skolemize

∃x∃yR(x, y) ⟹ R(c,d)

∀x∀y∃z(R(x, y) → P(z)) ⟹ z = f(x,y) ∀x∀y(R(x, y) → P(f(x,y))

POSSIBLE QUESTION IN EXAM

Suppose I have a language L = (P = {P,R}, F ={f,c}, α )

α (P) = 1 α (R) = 2 α (f) = 1 α (c) = 0

Are these terms:

f(x) yes, it is a term

P(x) no, is a formula

f(x,y) no, f has arity 1, not two

Show the free and bound occurrences of variables in

(∀y(R(y,x)) → ¬ P(f(y))

α (P) = 1 α (R) = 2 α (f) = 1 α (c) = 0

Bound and free

y in (∀y(R(y,x)) is bounded

x in (∀y(R(y,x)) is free

y in ¬ P(f(y)) is free

∀x(R(x,f(x)) → R(x,y))

Bound and Free

x in (R(x,f(x)) bound

f(x) in (R(x,f(x)) bound

x in R(x,y) bound

y in R(x,y) free

Give me two examples of a ground term in L

Ground term:

Constant c is correct

f(x) or f(f(c)) or f(f(f(c)))… are correct

x, y f(x) are ground or not

Sentence

A formula where no variable occurs free

∀xR(x,y) ∀x(R(x,f(x))

y is free, so ∀xR(x,y) is not a sentence

∀x(R(x,f(x)) is a sentence

∀xR(x,x) → P(x)

x in P(x) is free